THE INFLUENCE OF THE SPIRAL BEVEL GEARS GEOMETRY ON THE GLOBAL GEOMETRIC FACTOR

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Abstract: For the contact calculus, the normal force between the teeth is considered to be applied in a point between the inner point of single tooth contact and the mid-point of the zone of contact, depending on the overlap ratio $\varepsilon_{v\beta}$. In this paper the authors propose the introduction of a new factor in the calculus relation of the spiral bevel gears, named global geometric factor for contact stress Z_g, which includes the factors that take into consideration the geometry of the spiral bevel gear. The analysis of this factor is performed considering the main geometrical parameters of the gear.

1. INTRODUCTION

The results of the numerous researches accomplished on international level in the field of bevel gears with intersecting axes directed to the elaboration of international specifications (ISO 10300-1, 2 [9, 10], ANSI/AGMA 2003-B97 [7], respectively DIN 3991 [8]), which standardize the stress calculation of these gears. These specifications take into consideration a great number of functional and constructive factors which influence the endurance and the resistance of the gear at the contact stress, but also at the bending stress.
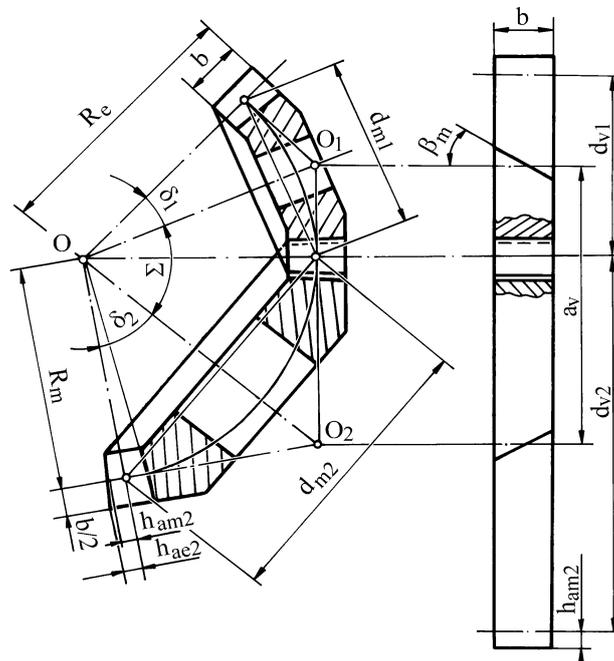

Fig. 1. Spiral bevel gear and substitute helical gear

Compared to the cylindrical gears, in which most often the contact between the teeth is linear, in spiral bevel gears the contact between the teeth takes place on elliptically shaped pressure surface, due to the bulging out of the teeth, both on their length and on their height [2, 3, 4, 5]. The maximal contact stress of any gear appears during the single pair gearing process. In the case of spiral bevel gears, the contact stress calculus is performed on the medium teeth section, applying the assumptions and the calculus

relations from helical gears on the virtual substitute gear corresponding to the medium frontal cones of the real bevel gear [5, 10].

The substitute gear of a spiral bevel gear (Figure 1) is a helical gear with the following features [1, 5]:

- the helix angle of the virtual helical gear is equal with the helix angle β_m of the spiral bevel gears;
- the pitch radii $d_{v1,2}/2$ are equal with the length of the two medium frontal cones generators;
- the normal module of the substitute gear is equal with the mean normal module of the spiral bevel gear;
- the tooth height of the substitute gear is equal with the mean tooth height of the spiral bevel gear;
- the tangential force at the pinion of the substitute gear is equal with the tangential force in the mean section of the spiral bevel gear.

The contact calculus, accomplished on the virtual cylindrical helical gears, is realized through the correction of the relation determined on a theoretical calculus model, with factors which take into consideration the differences that appear between the real gear and the calculus model. The calculus factors consider, on the one side, the bevel gears geometry and, on the other side, the construction of the gears and of the transmission the gear is part of (shafts, case), the mesh stiffness, shafts and case, the technology and the accuracy grade of bevel gears. In this paper, the influence of the bevel gears geometry on the gear overall size is analyzed.

The relation for the **contact stress** is based on the relation of Hertz and considers that the load is distributed on the length of middle line of contact. The position of the application point of the normal force between the teeth takes into consideration the overlap ratio $\varepsilon_{\upsilon\beta}$ and it is between the inner point of single tooth contact B and the mid-point of the zone of contact M (Figure 2) as following [7, 8, 10]:
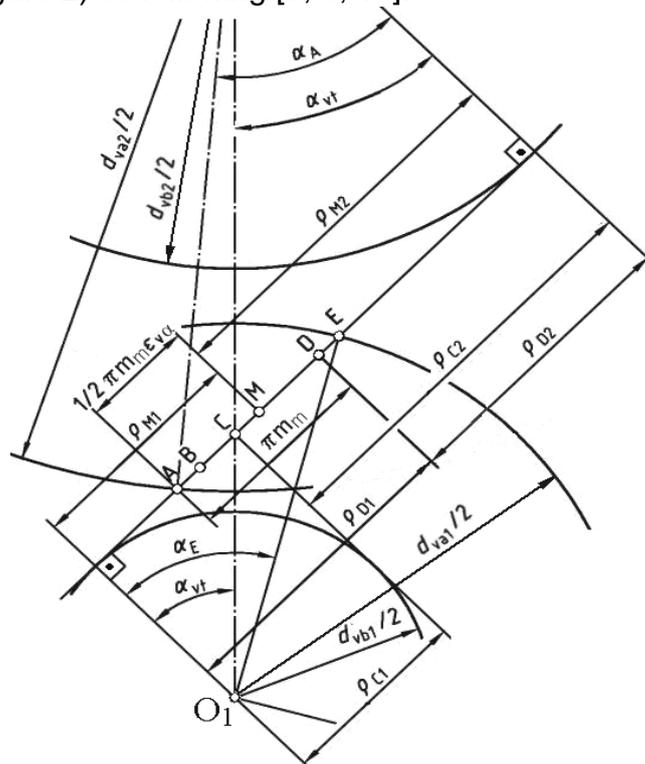
Fig. 2. Radius of curvature of tooth profile for characteristic points

- in the inner limit of single tooth contact B, for the gears with $\varepsilon_{\upsilon\beta}=0$;
- in the mid-point of the zone of contact M, for the gears with $\varepsilon_{\upsilon\beta} >1.0$;
- in the point obtained through interpolation of the above presented cases, for the gears with $0< \varepsilon_{\upsilon\beta} <1.0$.

2. THEORETICAL CONSIDERATIONS

The contact stress is determined with the relation [8, 10]

$$\sigma_H = \sqrt{\frac{F_{mt}K_A K_\upsilon K_{H\beta} K_{H\alpha}}{d_{m1} l_{bm}} \frac{\sqrt{u^2 +1}}{u}} Z_{M-B} Z_H Z_E Z_{LS} Z_\beta Z_K \leq \sigma_{HP},\qquad(1)$$

in which F_{mt} is the nominal tangential force at reference cone at mid-face width; d_{m1} – mean pitch diameter of the bevel pinion; l_{bm} – length of middle line of contact; u – gear ratio of bevel gear; Z_{M-B} – mid-zone factor; Z_H – zone factor; Z_E – elasticity factor; Z_{LS} –load sharing factor; Z_β – helix angle factor for contact stress; Z_K – bevel gear factor (flank); K_A – application factor; K_υ – dynamic factor; $K_{H\beta}$ – face load factor for contact stress; $K_{H\alpha}$ – transverse load factor for contact stress.

The force F_{mt} is expressed depending on the pinion torque T_1, $F_{mt} = 2T_1/d_{m1}$, and the mean diameter of the bevel pinion depending on the mean cone R_m and the gear ratio u, resulting

$$d_{m1} = 2R_m \sin\delta_1 = \frac{2R_m}{\sqrt{u^2 +1}}.\qquad(2)$$

The length of middle line of contact is determined depending on the value of the transverse contact ratio $\varepsilon_{\upsilon\alpha}$, the overload ratio $\varepsilon_{\upsilon\beta}$, respectively the modified contact ratio $\varepsilon_{\upsilon\gamma}$:

$$l_{bm} = \frac{b\varepsilon_{\upsilon\alpha}}{\cos\beta_{\upsilon b}} \frac{\sqrt{\varepsilon_{\upsilon\gamma}^2 - \left[(2-\varepsilon_{\upsilon\alpha})(1-\varepsilon_{\upsilon\beta})\right]^2}}{\varepsilon_{\upsilon\gamma}^2},\qquad(3)$$

if $\varepsilon_{\upsilon\beta} <1.0$;

$$l_{bm} = \frac{b\varepsilon_{\upsilon\alpha}}{\varepsilon_{\upsilon\gamma}\cos\beta_{\upsilon b}},\qquad(4)$$

if $\varepsilon_{\upsilon\beta} \geq 1.0$.

The factor Z_{M-B} is determined [6, 10] with the relation

$$Z_{M-B} = \frac{\tan\alpha_{\upsilon t}}{\sqrt{\left[\sqrt{\left(\dfrac{d_{\upsilon a1}}{d_{\upsilon b1}}\right)^2 -1 - F_1 \dfrac{\pi}{z_{\upsilon 1}}\right]\left[\sqrt{\left(\dfrac{d_{\upsilon a2}}{d_{\upsilon b2}}\right)^2 -1 - F_2 \dfrac{\pi}{z_{\upsilon 2}}\right]}}\qquad(5)$$

in which $\alpha_{\upsilon t}$ is the transverse pressure angle of virtual cylindrical gear; $d_{\upsilon a1,2}$ – the tip diameters of virtual cylindrical gears; $d_{\upsilon b1,2}$ – base diameters of virtual cylindrical gears; $z_{\upsilon 1,2}$ – the numbers of teeth of the virtual gears; $F_{1,2}$ – auxiliary factors for mid-zone factor.

For the calculation of the auxiliary factors F_1 and F_2, the following relations [9] are used:

$$F_1 = 2 + (\varepsilon_{\upsilon\alpha} -2)\varepsilon_{\upsilon\beta};\ F_2 = 2\varepsilon_{\upsilon\alpha} -2 + (2-\varepsilon_{\upsilon\alpha})\varepsilon_{\upsilon\beta},\qquad(6)$$

for $0< \varepsilon_{\upsilon\beta} \leq 1$, respectively $F_1 = F_2 = \varepsilon_{\upsilon\alpha}$, for $\varepsilon_{\upsilon\beta}>1$, in which $\varepsilon_{\upsilon\alpha}$ is the transverse contact ratio of virtual cylindrical gear; $\varepsilon_{\upsilon\beta}$ – overlap ratio of virtual cylindrical gear.

The zone factor for contact stress Z_H is determined [6, 10] with the relation

$$Z_H = 2\sqrt{\frac{\cos\beta_m}{\sin(2\alpha_{\upsilon t})}}\,,\qquad\qquad\qquad (7)$$

and the helix angle factor for contact stress, with the relation

$$Z_\beta = \sqrt{\cos\beta_m}\,.\qquad\qquad\qquad (8)$$

The relations (6) were determined through interpolation between the straight bevel gear with $\varepsilon_{\upsilon\beta} = 0$ and the spiral bevel gear with $\varepsilon_{\upsilon\beta}= 1$ [9]. The relations to determine the geometrical elements which appear in the calculus expressions, presented above, are given in the professional literature [8, 9, 10] and are defined depending on the mean normal module of the teeth, the purpose being the extension of the obtained results.

Introducing the relation (2) in relation (1), it results

$$\sigma_H = Z_E Z_{LS} Z_K Z_g \sqrt{\frac{T_1 K_A K_\upsilon K_{H\beta} K_{H\alpha}}{2R_m^2}} \le \sigma_{HP}\,,\qquad\qquad (9)$$

where Z_g represents the global geometric factor for the contact stress of spiral bevel gears. This factor, defined for the straight bevel gear in [1], is proposed by the authors for the spiral bevel gears which it is determined with the relation

$$Z_g = \sqrt{\frac{1}{I_{bm}}\frac{\left(u^2+1\right)^{3/2}}{u}Z_{M-B}Z_H Z_\beta}\,.\qquad\qquad (10)$$

It can be noticed that the factor Z_g takes into consideration the teeth geometry and the factors Z_E and Z_K have constant values and do not influence the value of the factor Z_g.

3. CALCULUS AND CONCLUSIONS

Based on the above presented calculus relations, there was elaborated a computational program to determine the influence of the geometrical parameters of the spiral bevel gear on the global geometric factor, the values of which influence the contact stress. The main analyzed parameters are: the number of teeth of the pinion z_1, the gear ratio u, the mean spiral angle β_m, the profile shift coefficient x_{hm1}, respecting the condition $x_{hm2}=-x_{hm1}$; the width coefficient of the gears ψ_{Rm}; it was considered that the spiral bevel gears teeth have the pressure angle $\alpha_n=20°$, the addendum related to m_{mn}, $h_{an}^*=1$ and the dedendum related to m_{mn}, $h_f^*=1.25$.

The diagram presented in Figure 3 indicates the variation of the global geometric factor Z_g of the spiral bevel gear depending on the profile shift coefficient x_{hm1} and the number of teeth of the pinion z_1. The diagram shown in Figure 4 indicates the variation of the same factor depending on the gear u. The rest of the parameters have constant values, these are registered in the diagram.

The analysis of the variation curves of the global geometric factor for contact stress allows to point out the following conclusions:

- The geometric factor Z_g has a decreasing variation with the increase of the profile shift coefficient, regardless of teeth number z_1 of the bevel pinion and the gear ratio u;
- The geometric factor Z_g also decreases with the increase of the teeth number of the pinion, excepting the cases when $z_1=7$, respectively 8 teeth, when the variation curves have a bigger reducing slope, intersecting some of the other variation curves;
- The factor Z_g diminishes with the increase of the teeth number z_1 of the bevel pinion, but this factor increases with the growth of the gear ratio u;

2.64

- The variation limits of the range of factor Z_g are larger in the case of the gear ratio variation than in the case of the pinion teeth number variation.
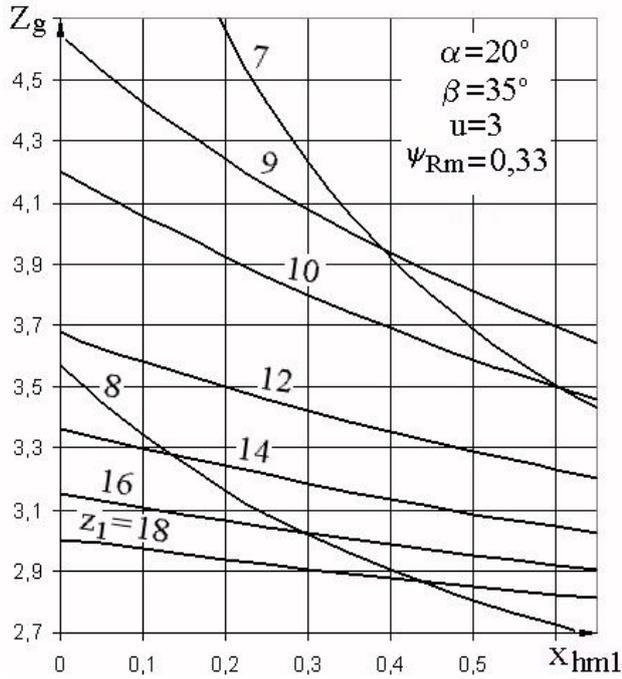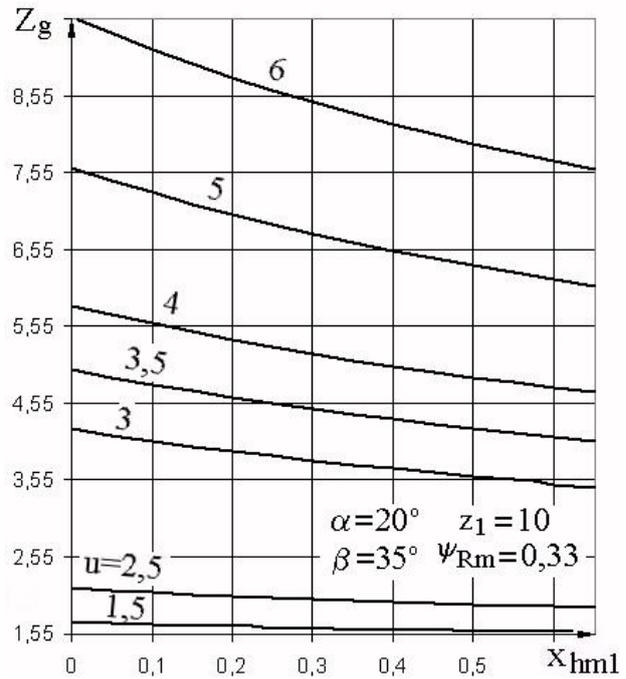

Fig. 3. $Z_g=f(x_{hm1}, z_1)$

Fig. 4. $Z_g=f(x_{hm1}, u)$

The influence of the mean spiral angle β_m, respectively of the width coefficient ψ_{Rm}, is presented in Figure 5, respectively in Figure 6. The rest of the parameters have constant values, registered in the diagram.
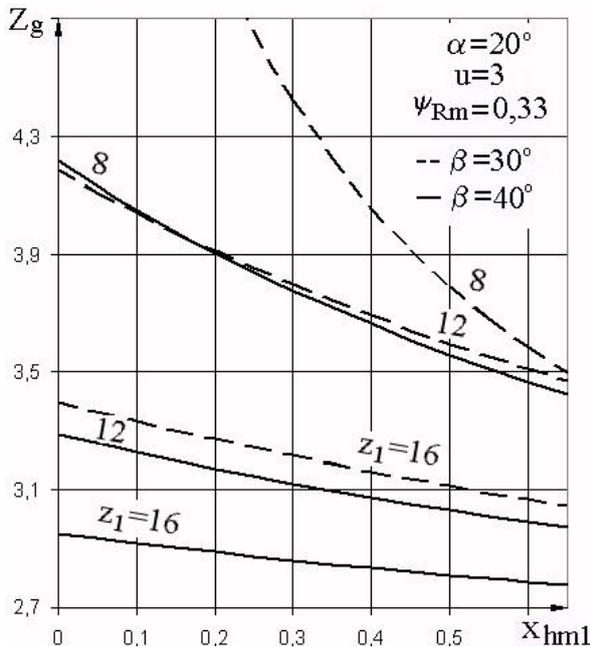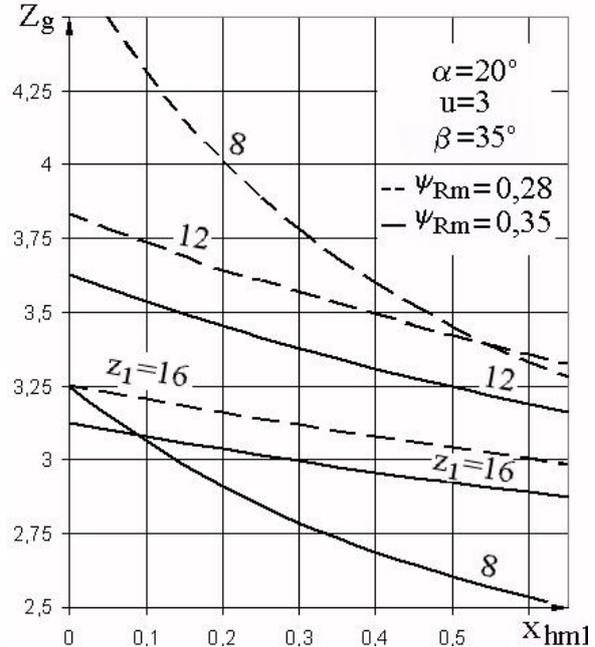

Fig. 5. $Z_g=f(x_{hm1}, z_1, \beta_m)$

Fig. 5. $Z_g=f(x_{hm1}, z_1, \psi_{Rm})$

The analysis of the variation curves of the global geometric factor for contact stress allows to point out the following conclusions:

- As expected, the value of the geometric Z_g decreases with the increase of the mean spiral β_m and the increase the profile shift x_{hm1};
- The decrease of the pinion teeth number z_1 leads to the increase of the value of factor Z_g, regardless of the value of angle β_m;

- Regarding the influence of the gear width coefficient, it can be asserted that the geometric factor Z_g has a bigger value at reduced values of the width coefficient, respectively a lower value at higher values of the width coefficient.

Concluding, through the proper selection of the main geometrical parameters of a spiral bevel gear, there can be obtained a gear with reduced overall size, which can transmit an assessed torque, at an assessed rotational speed, in the conditions of maximum use of the mechanical properties of the materials the bevel gears are made of.

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